PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Magnetisation of Iron and Other Magnetic Metals in Very Strong Fields

J. A. Ewing and William Low

Phil. Trans. R. Soc. Lond. A 1889 **180**, 221-244
doi: 10.1098/rsta.1889.0007

VII. *On the Magnetisation of Iron and other Magnetic Metals in very Strong Fields.*

By J. A. Ewing, *B.Sc., F.R.S., Professor of Engineering in University College, Dundee, and* William Low.

Received October 29,—Read November 22, 1888.

§ 1. Early in 1887 we communicated to the Royal Society a short account of experiments made to examine the magnetic behaviour of iron when subjected to strong magnetic force by what we called the "isthmus" method of magnetisation.* Since then the experiments have been continued and extended by applying stronger magnetic forces, and by testing samples of nickel, cobalt, and various steels, as well as wrought iron and cast iron.† It may be well to preface an account of the more recent experiments by a short summary of the results stated in our earlier paper.

§ 2. The method of experiment consisted in turning the piece of metal whose magnetisation was to be examined to the form of a bobbin with a narrow neck or isthmus, and placing that between the pole-pieces of a powerful electromagnet. The sample was furnished with a spreading cone at each end, to facilitate the convergence of the lines of magnetic induction upon the central neck. The magnetisation was measured by means of an induction coil of fine wire wound in a single layer, or, in some cases, in two layers, upon the metal of the neck. Outside of this coil, and at a small definite distance from it, a second induction coil was wound in order to measure the magnetic field in the space between the two coils. This served a double purpose: it enabled a proper deduction to be made from the values of the induction measured within the inner coil, to allow for the space between the surface of the iron neck and the centre of the thickness of the coil; and it gave values of the magnetic force in the space immediately surrounding the iron, from which an inference might be drawn as to the value of the force within the neck itself. As there was no free magnetism on the circumference of the neck, in the medial plane, the force within the metal was continuous there with the force outside, and it will be shown later that when a suitable slope was given to the conical ends of the bobbin the variation of force in the medial plane in directions at right angles to the axis was so small that the external field

---

* "On the Magnetisation of Iron in Strong Fields," 'Roy. Soc. Proc.,' vol. 42, p. 200.

† Preliminary notices of some of the later results were communicated to Section A of the British Association at Manchester ('Report of the British Association for 1887,' pp. 586 and 587).

must have formed (in such cases) a very approximately accurate measure of the force acting on the metal. In other cases, when the cones were more blunt, the force in the external field was somewhat greater than the mean force within the metal.

§ 3. Figs. 1 and 2, copied from our earlier paper, show the forms of bobbin originally used, and the pole-pieces of the magnet by which they were magnetised. With bobbins of the type of fig. 1, the magnetic induction in the neck and the field in the surrounding air were measured by suddenly turning the bobbin round, end for end; in bobbins of the type of fig. 2, the measurements were made by suddenly withdrawing the bobbin from its place between the pole-pieces. In the latter case, the induction measured was the excess of the whole induction ($\mathfrak{B}$) above the residual induction ($\mathfrak{B}_r$) which persisted when the bobbin was drawn out. In iron bobbins the residual magnetisation was found to be sensibly constant from the lowest to the highest value of the inducing field employed in these experiments, but the form of the bobbin made the amount of this residue small. It was measured in bobbins of the type of fig. 1, by comparing the result of turning the bobbin round with the effect of drawing the bobbin out; and, in the first instance, its value in iron bobbins of the type of fig. 2, was estimated to be about the same as in bobbins of the type of fig. 1. In later experiments, when other more retentive metals were being examined, and the residual magnetism consequently formed a more important part of the whole, its value was directly determined by using built-up bobbins which allowed one conical end to be withdrawn; the residual magnetism was then determined after the bobbin had been removed from the field by slipping off (in one operation) one of the conical ends, along with an induction coil which had been wound for this purpose upon a loose ring over the central neck.

Fig. 1.

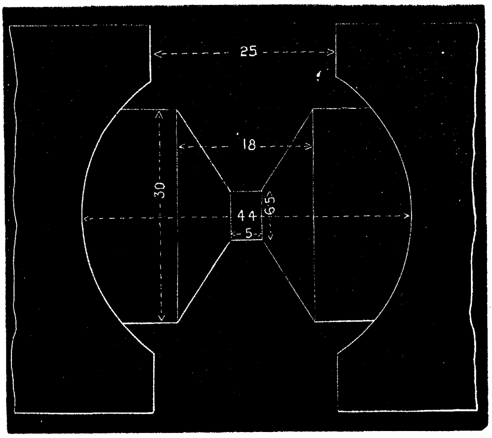
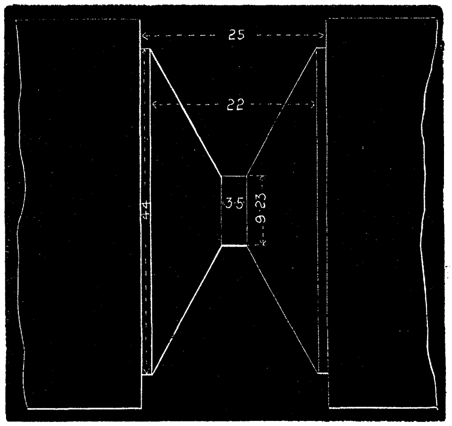


Fig. 2.



Wrought Iron.

§ 4. In the early experiments solid bobbins of the form and dimensions shown in fig. 1, were tested, one of Lowmoor, and another of Swedish wrought iron, with necks 6·5 mm. in diameter and 5 mm. long. The magnetic force was measured in an annular space between the inner and outer induction coils, about 1·3 mm. wide and closely contiguous to the iron neck: for brevity we shall call the magnetic force thus measured in the surrounding air space the "outside field." Tables I. and II. below, which are copied for convenience of reference from our earlier paper, give observed values of the induction $\mathfrak{B}$, and of the outside field for various strengths of current in the coils of the field magnets. They also give values of the quantity ($\mathfrak{B}$ − outside field)/4π, which would be a measure of the intensity of magnetisation $\mathfrak{I}$ if the outside field were fairly representative of the mean magnetic force within the metal of the neck itself (since $\mathfrak{B} = 4\pi\mathfrak{I} + \mathfrak{H}$); and also of the quantity $\mathfrak{B}$/outside field, which on the same proviso would measure the permeability μ. The residual induction $\mathfrak{B}_r$ was 510 in the Lowmoor and 500 in the Swedish sample. The magnetic quantities are stated in c.g.s. units.

TABLE I.—Lowmoor Wrought Iron.

Current in field magnets, ampères.	Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1·98	3,630	24,700	1680	6·80
4·04	6,680	27,610	1670	4·13
5·81	7,800	28,870	1680	3·70
7·60	8,810	29,350	1630	3·33
11·0	9,500	30,200	1650	3·18
13·5	9,780	30,680	1660	3·14
16·2	10,360	30,830	1630	2·98
21·6	10,840	31,370	1630	2·89
26·8	11,180	31,560	1620	2·82

TABLE II.—Swedish Wrought Iron.

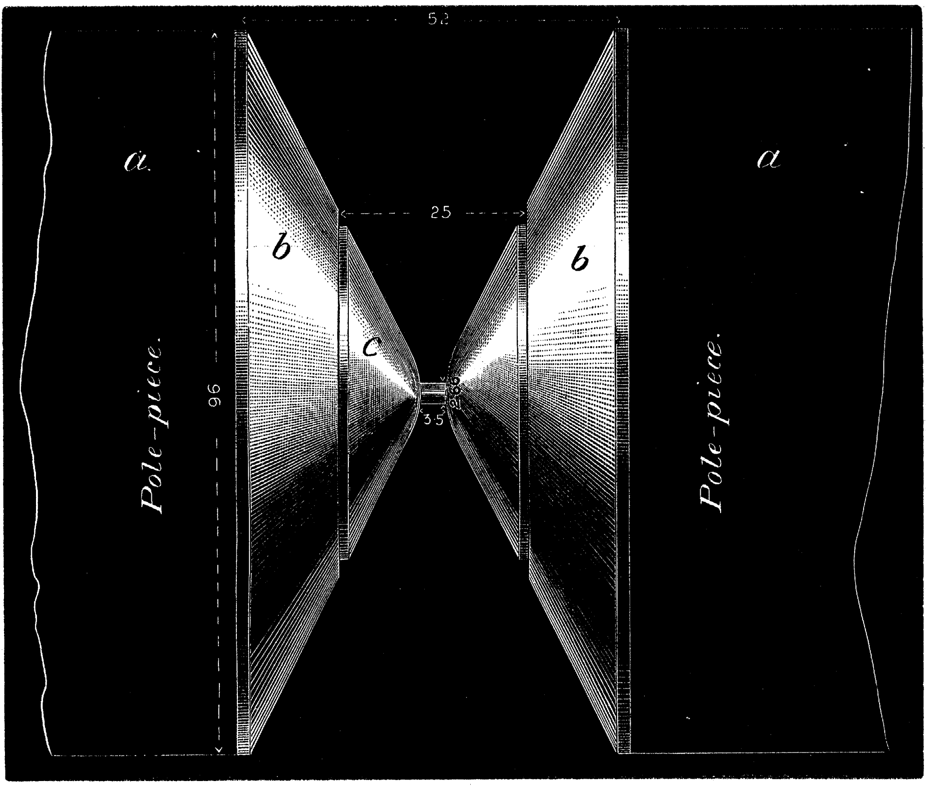
Current in field magnets, ampères.	Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
4·08	6,690	27,960	1700	4·18
7·77	8,900	29,730	1660	3·34
10·9	9,510	30,820	1700	3·24
14·2	10,000	31,210	1690	3·12
16·5	10,360	31,630	1700	3·05
18·9	10,810	31,720	1670	2·94
22·9	10,880	32,060	1690	2·95
26·5	11,200	32,360	1690	2·90

Results closely accordant with these were also obtained with samples of the form shown in fig. 2; and, as bobbins with flat ends were most convenient, especially in very strong fields, the subsequent experiments were all made with the flat-ended type.

§ 5. The pole-pieces of the magnet used in the early experiments were only 5·25 cm. square, and it was clear that with a larger magnet a greater concentration of the induction might be produced. Professor TAIT was kind enough to lend us the large magnet of the Edinburgh University Laboratory, and all the subsequent work has been done with it.

The Edinburgh magnet has a pair of vertical limbs about 50 cm. long and 10·7 cm. in diameter. These are united by a horizontal yoke at the bottom, and are furnished on the top with movable pole-pieces in the form of rectangular blocks of soft wrought iron, the cross-section of which is 9·6 cm. square. The limbs are wound along a length of 49 cm. with a number of coils which are grouped in series, making about 1600 turns in all. The currents employed by us ranged up to 40 ampères, and the greatest value of the line-integral of the magnetic force was consequently about 80,000. To allow the old bobbin, of the form of fig. 1, to be effectively used, we furnished the magnet poles with a pair of intermediate conical pieces of soft wrought iron, which virtually formed an extension of the bobbin's ends. Fig. 3 is a full-size sketch of the poles a, a, with the intermediate pieces b, b, and the bobbin c, in place. The figure shows the size to which the neck of the bobbin was finally turned down in experiments which are described below. The dimensions are given in millimetres.

Fig. 3.



§ 6. In the first instance, however, a Lowmoor bobbin of the dimensions shown in fig. 1, which had been used in the earlier observations, was tested in the Edinburgh magnet without being turned down. The following are the particulars of this experiment and the results :—

Diameter of central neck, 9·23 mm. Length of neck, 3·5 mm.

Diameter to middle of inner induction coil, 9·48 mm.

Diameter to middle of outer induction coil, 10·99 mm.

Inner induction coil, a single layer of twelve turns of silk-covered wire, 0·25 mm. in diameter over the silk.

Outer induction coil, a single layer of seven turns of the same wire.

TABLE III.—Lowmoor Wrought Iron.

Current in field magnets, ampères.	Outside field.	𝔅.	$\frac{𝔅 - \text{outside field}}{4\pi}$.	$\frac{𝔅}{\text{outside field}}$.
15·0	15,990	36,460	1630	2·28
18·5	18,410	36,970	1480	2·01
28·5	18,380	37,320	1510	2·04
33·0	18,570	37,610	1520	2·03
38·5	18,900	37,990	1520	2·01

A Swedish iron sample of the same shape gave an induction of 37,620 in a field of about the same force, with a current of 38 ampères in the field magnet coils.

§ 7. To push the induction in the Lowmoor sample to a still higher value, the neck of the bobbin was turned down to a diameter of 3·97 mm., the slope of the conical ends being approximately maintained. The inner coil was re-wound with a mean diameter of 4·22 mm., and the outer coil with a mean diameter of 5·7 mm. The following are the results for a magnetising current of about 38 ampères, the mean of several measurements being taken :—

Outside field.	𝔅.	$\frac{𝔅 - \text{outside field}}{4\pi}$.	$\frac{𝔅}{\text{outside field}}$.
25,620	43,500	1430	1·7

§ 8. A final effort to raise the induction was then made by again turning down the neck of the sample to the size shown in fig. 3. The diameter of the neck was reduced to 2·66 mm., but its length was not reduced in the same proportion: to leave room for a sufficiently long induction coil, a little of the metal of the cones close to the neck was cleared away in the manner shown by the sketch. The section of the neck was now less than $\frac{1}{1500}$ that of the pole-pieces. The bobbin was annealed after being turned down. The inner induction coil was wound in a single layer of ten turns with a mean diameter of 2·93 mm. The outer coil was a single layer of eight turns with a mean diameter of 4·36 mm. With these conditions the induction was forced to the enormous value of 45,350 c.g.s. units, though the outside field between the two coils had a somewhat smaller value than before. This anomaly does not necessarily imply that the measurements were in error, for, as will appear from what follows, the relation of the outside field to the force within the metal is materially affected by the form of the conical ends, and that form had been altered, as has just been said, in the region close to the neck. The excessive smallness of the neck in this case, however, made it more difficult than before to measure the outside field with precision. The following are mean results for the strongest magnetising currents :—

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
24,500	45,350	1660	1·85

§ 9. With regard to the quantity ($\mathfrak{B}$ — outside field)/4π, it will be noticed that, if we exclude the last (somewhat doubtful) case, there is a progressive decrease as the induction rises, within the range covered by these experiments. With a field of 5000 or 6000, the value of this quantity was 1700 in the Swedish sample and 1680 in the Lowmoor sample, and it fell to 1430 as the field was raised to 25,000. This gives great interest to the question, whether the field as measured in the outside space has the same, or nearly the same, value as the magnetic face within the metal; for in that case we should have evidence that the intensity of magnetisation $\mathfrak{I}$ passes a maximum and begins to decrease under the action of very strong fields, and this is a result which WEBER's molecular-current theory of diamagnetism, extended as MAXWELL has extended it to a paramagnetic substance, would lead us to expect.* After a careful examination of this important point, we have concluded, for reasons given below, that the apparent decrease of $\mathfrak{I}$ in the experiments described above is in all probability wholly due to the outside field being greater than the field within the metal, and that, if there is any variation in the real value of $\mathfrak{I}$ in strong fields, it is smaller than our method of experiment can detect.

§ 10. An attempt to investigate the uniformity of the field (in a medial plane along lines radiating from the axis) was made by building up a bobbin over the neck of which four induction coils were wound, one above another, with small annular spaces between. The lowest coil was wound on the iron neck, and the other three on thin

---

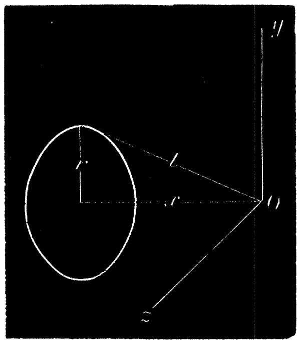
* See MAXWELL's 'Treatise on Electricity and Magnetism,' vol. 2, chap. 22:—"If it should ever be experimentally proved that the temporary magnetisation of any substance first increases and then diminishes as the magnetising force is continually increased, the evidence of the existence of these molecular currents would, I think, be raised almost to the rank of a demonstration."

brass tubes turned to slip one on another, with flanges at the ends to preserve a definite clearance between them and to keep them concentric. The innermost coil had a mean diameter of 3·2 mm., and the outermost a mean diameter of 7·8 cm. The annular space between them, 2·3 mm. thick, was in this way divided into three parts, in each of which the field was measured. It was found that in these three parts the field decreased progressively with increase of distance from the axis. Thus, in one instance, the field fell from 19,000 in the innermost ring to 17,300 in the outermost ring. It is unnecessary to describe at length these experiments, which were very laborious, and which did not throw any light on the important question of how closely the force within the metal approximated to the force in the air close to the surface of the neck. Moreover, the form of the built-up bobbin used in this case was, as we afterwards recognised, such as to give a much more uniform field than the bobbin formerly used.

Concentration of Magnetic Force by Conical Pole-faces.

§ 11. The magnetic force in the space between the pole-pieces is made up of two parts: (1) the electromagnetic force directly produced by the current in the field-magnet coils; and (2) the force due to free magnetism distributed for the most part over the pole-faces. The first of these was a comparatively small part (less than one-fiftieth) of the whole, and its value must have been sensibly uniform at such small distances from the axis as those with which we are now concerned. In considering the uniformity of the field we need only deal with the force produced by free magnetism distributed over the opposing surfaces of the poles.

Fig. 4.



§ 12. The free magnetism on each pole-face may be treated as made up of a series of co-axial circular rings in planes normal to the axis of magnetisation. Calling M the whole free magnetism of one of these rings (fig. 4) and r its radius, the force F due to it at a point in the axis at a distance x from the plane of the ring is $\mathrm{M}x/l^3$, where $l = \sqrt{(r^2 + x^2)}$. This force will be a maximum when $d\mathrm{F}/dx = 0$, or

$$\frac{1}{l^3} - \frac{3x^2}{l^5} = 0,$$

which gives

$$x = \frac{r}{\sqrt{2}}; \qquad \tan\theta = \sqrt{2}; \qquad \theta = 54^\circ\,44'.$$

Hence, a series of co-axial rings will be most advantageously disposed for producing force at a point on the axis if they lie on a cone having its vertex at that point, with a semi-vertical angle of 54° 44′. This conclusion is independent of the distribution of density from ring to ring.

§ 13. The greatest force will be produced when the pole-pieces are themselves saturated, so that $\mathfrak{I}$ reaches its limiting value in all parts of the metal. In that case the distribution of density from ring to ring is uniform. The surface density of free magnetism at any point of a sloping pole-face is $\mathfrak{I}\sin\theta$, where θ is the slope of the face to the axis of magnetisation. The whole quantity in each ring is $\mathfrak{I}$ multiplied by the area of the ring projected upon a plane normal to the axis—a quantity which is independent of the slope of the cone. We have, therefore, the same series of attracting rings to deal with, whatever be the slope of the convergent forces, and whether that slope be uniform or not. Given, then, a certain diameter for the neck of the bobbin to be magnetised, the greatest magnetic force will be produced at the middle of the axis of the neck if we make the expanding ends and pole faces in the form of cones, with a semi-angle of 54° 44′, and with their vertices at the middle of the neck.*

§ 14. This cone of maximum concentrative power is not, however, the form best suited for producing a uniform field. At any point in the axis $d\mathrm{F}/dy$ and $d\mathrm{F}/dz$ are zero, axes of y and z being taken in a plane parallel to the rings, and the condition for a uniform field (uniform, namely, in the neighbourhood of the axis, over a transverse plane) is that $d^2\mathrm{F}/dy^2$ and $d^2\mathrm{F}/dz^2$ shall also be zero. Consider again the attraction of a ring at any point O in the axis. Taking LAPLACE's equation—

$$\frac{d^2\mathrm{V}}{dx^2} + \frac{d^2\mathrm{V}}{dy^2} + \frac{d^2\mathrm{V}}{dz^2} = 0,$$

and differentiating with respect to x, we have

$$\frac{d^2}{dx^2}\frac{d\mathrm{V}}{dx} + \frac{d^2}{dy^2}\frac{d\mathrm{V}}{dx} + \frac{d^2}{dz^2}\frac{d\mathrm{V}}{dx} = 0.$$

By symmetry of the field about the axis of x the second and third terms are equal; hence, writing F for $d\mathrm{V}/dx$,

$$\frac{d^2\mathrm{F}}{dx^2} + \frac{2d^2\mathrm{F}}{dy^2} = 0.$$

---

* The corresponding proposition for truncated cones, with an air space between them, has lately been stated by Professor STEFAN ('Wien, Akad. Sitzber.,' Feb. 9, 1888; or 'Phil. Mag.,' vol. 25, p. 322).

The condition for a uniform field will therefore be satisfied when $d^2\mathrm{F}/dx^2 = 0$; that is, when

$$-\frac{9x}{l^5} + \frac{15x^3}{l^7} = 0,$$

which gives $x = r\sqrt{\tfrac{3}{2}}$; $\tan\theta = \sqrt{\tfrac{2}{3}}$; $\theta = 39° \, 14'$. Thus, the condition is satisfied when the pole-faces are cones converging as before upon the middle of the neck, but with a semi-vertical angle of 39° 14′.*

§ 15. With a cone of any semi-angle θ, the surface density of free magnetism being $\mathfrak{I}\sin\theta$, the force at the vertex due to a ring at an axial distance x, of radius r, and of length dl, measured along the slope, is

$$2\pi r\, dl.\ \mathfrak{I}\sin\theta.\ x/l^3, \quad \text{or} \quad 2\pi\mathfrak{I}\sin^2\theta\cos\theta\, dr/r.$$

The whole force at the vertex is

$$2\pi\sin^2\theta\cos\theta\int_a^b \frac{\mathfrak{I}dr}{r},$$

a being the radius of the neck on which the cone converges, and b the radius of the base to which it spreads.

Hence (treating $\mathfrak{I}$ as uniform), with a pair of truncated cones, joined by a neck at the middle of which they have their common vertex, the whole force there is

$$\mathrm{F} = 4\pi\mathfrak{I}\sin^2\theta\cos\theta\log_\epsilon\frac{b}{a},$$

which, for convenience of calculation, we shall write

$$\mathrm{F} = 28{\cdot}935\,\mathfrak{I}\sin^2\theta\cos\theta\log_{10}\frac{b}{a}.$$

§ 16. Applying this to the cones of maximum concentrative power (§ 13), in which $\sin\theta = \sqrt{\tfrac{2}{3}}$ and $\cos\theta = \frac{1}{\sqrt{3}}$,

$$\mathrm{F}_{max.} = 11{\cdot}137\,\mathfrak{I}\log_{10}\frac{b}{a},$$

and the greatest value of the force will be obtained when $\mathfrak{I}$ has the saturation value (of say 1700 c.g.s. units for soft wrought iron), in which case

$$\mathrm{F}_{max.} = 18930\log_{10}\frac{b}{a},$$

an expression which measures the greatest possible force which the "isthmus" method of magnetisation can apply at a point in the axis of the bobbin (over and above the small force which is directly produced by the magnet coils). It is

---

* We are indebted to Mr. A. TANAKADATE for suggesting this calculation of the form of poles suited to give a uniform field.

impracticable to produce quite so great a force as this on account of the difficulty of saturating the magnet poles.

§ 17. With the cones which give the most uniform field, for which $\sin\theta = \sqrt{\tfrac{2}{5}}$ and $\cos\theta = \sqrt{\tfrac{3}{5}}$, the value of F is only

$$8{\cdot}965\ \mathfrak{I}\log_{10}\frac{b}{a},$$

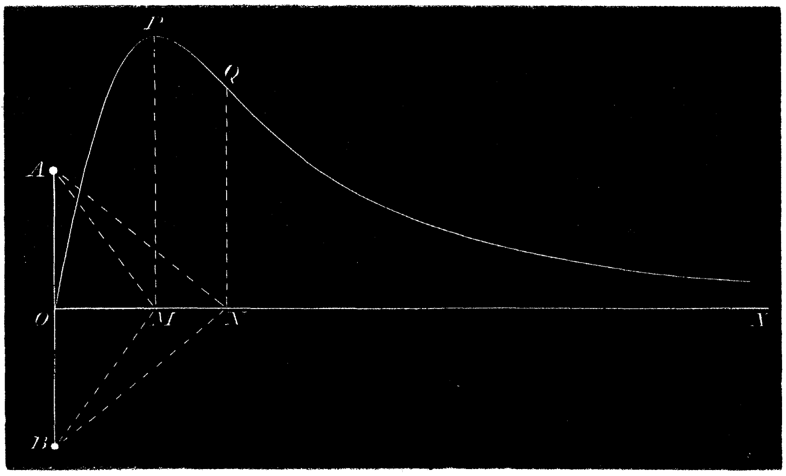
which becomes

$$15240\log_{10}\frac{b}{a}$$

in the event of the pole-pieces being of soft wrought iron and saturated.

§ 18. The curve, fig. 5, has been drawn to show how the force at the vertex varies when the angle of the cone is altered.

Fig. 5.



The base of the cone being represented by AB, any ordinate PM gives the force when the vertex is at M. In the figure, AMB represents the cone of maximum concentrative power, and ANB represents the cone giving a uniform field in the neighbourhood of the axis, Q being the point of inflection in the curve.

§ 19. In figs. 6 and 7 the same two cases are further illustrated by curves which show the sum of the forces due to two equal and opposite rings (situated on cones with a common vertex) at points along the axis.

By summing up the effects of such pairs, for the whole cone, we may judge how nearly the force is uniform from end to end of the neck of the magnetised bobbin. In a bobbin with cones of semi-angle 39° 14′ the field is sensibly uniform from end to end of the neck, except close to the ends, where it is slightly reduced, and (§ 14) this longitudinal uniformity implies transverse uniformity.

§ 20. When the semi-vertical angle of the cones is greater than 39° 14′, the force at points on the axis has a maximum at the common vertex, and, since $d^2\mathrm{F}/dx^2$ and $d^2\mathrm{F}/dy^2$ have opposite signs (§ 14), the field is stronger at places near the axis than on the axis itself. In a bobbin with a narrow neck this may have the effect of making the field in the closely surrounding air space greater than the mean field within the neck.

§ 21. We may now apply the above conclusions to elucidate the experiments which have been described. The form of bobbin used in them had been chosen, without reference to theory, as one likely to give a strong concentration of magnetic induction, and it chanced to come very near the best form for this purpose. The cones had a semi-angle of 60°, and their vertices were nearly coincident (overlapping very slightly, see fig. 2).

Applying the formula of § 15, we have, for $\theta = 60°$,

$$F = 10{\cdot}85 \, \mathfrak{I} \log_{10} \frac{b}{a},$$

which is only two and a half per cent. short of the force attainable by using cones of maximum concentrative power. Moreover, it must be borne in mind that in actual use of the isthmus method the strongest induction will be reached when the semi-angle is rather *greater* than 54° 44′, for $\mathfrak{I}$ is itself a function of the angle, decreasing when the angle is decreased, on account of the augmented "resistance" of the whole magnetic circuit. For this reason we probably obtained as much concentration with cones of 60° as we should have obtained with cones of 54° 44′. Further, in the last experiment (§ 8), the neck of the bobbin had been turned down to the smallest size we found it practicable to work with. It is clear, therefore, that no materially higher value of $\mathfrak{B}$ than the value already obtained was possible with the apparatus at our disposal.

Fig. 6.

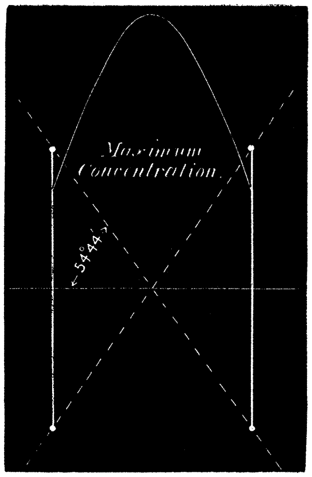
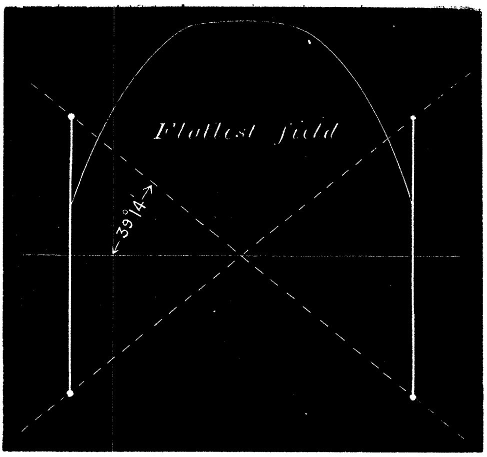


Fig. 7.



§ 22. From the relation of the line integral of magnetic force to the length of iron and mean length of air in the magnetic circuit, we estimate that the mean value of $\mathfrak{I}$

in the magnet cores and pole-pieces cannot have exceeded 1400 when the magnetising current was at its strongest. The distribution of this over the conical pole-faces was not uniform. Close to the neck it reached the saturation value (of, say, 1680 or 1700), being gathered there at the expense of outlying portions. This want of uniformity of $\mathfrak{I}$ in the pole-faces increases the magnetic force in the neck, but when a distribution of $\mathfrak{I}$ is assigned it is easily taken account of in applying the formula of § 15.

§ 23. Taking the experimental case (§ 7), in which the diameter of the neck was $\frac{1}{25}$ of the diameter to which the cones spread, we calculate that the magnetic force at the middle of the axis was probably about 22,500, and at other points of the axis it was less.

Now, the measured value of the field in the air, close to the bobbin's neck, was in this instance 25,620. To produce this force at the axis would require that the value of $\mathfrak{I}$ in the pole-pieces should have been nothing less than 1690 all over, that is to say, it would require that the poles should have been saturated from axis to circumference—a quite impossible supposition. It is clear that the force in the air close to the neck was in this case distinctly greater than the mean force within the neck.

The measured induction $\mathfrak{B}$ within the neck was 43,500. If we accept 22,500 as the mean value of $\mathfrak{H}$ within the neck (remembering that while $\mathfrak{H}$ increases from the axis to the circumference it diminishes from the middle towards the ends), the value of $\mathfrak{I}$ in the neck would be $(43{,}500 - 22{,}500)/4\pi = 1670$, which is, as nearly as may be judged, the same value as was produced by the application of magnetising forces of moderate strength.

§ 24. These considerations establish a very strong presumption that the apparent decrease of $\mathfrak{I}$ in the experiments, that is to say, the observed decrease in the quantity ($\mathfrak{B}$ — outside field)/4π under very strong forces is to be explained by the fact that the outside field was stronger than the field within the neck; and that the true value of $\mathfrak{I}$ is sensibly constant throughout the range of magnetic forces examined, namely, from about 4000 to 24,000 c.g.s. units.

Further Experiments on Wrought Iron.

§ 25. To put this matter further to the proof, we continued the experiments with another bobbin, also of Lowmoor wrought iron, the conical ends of which were shaped so as to produce a much more uniform field. The shape which would give the most uniform field was not chosen, for that would have imposed so low an upper limit on the strength of the field that the test would have been rather inconclusive. By way of compromise, a bobbin was turned of the shape and dimensions shown in fig. 8, with cones of semi-angle 45°, as a form which combined high concentrative power with a fair approximation to uniformity of field. The advantage, in respect of uniformity of field, which this bobbin had over the one formerly used may be judged from figs. 9 and 10, which show the longitudinal variation of force due to a pair of rings in the two cases. The length and diameter of the neck were 3·42 mm. The outside field and the induction were measured as before, and it was found that they were decidedly less than in the former instance, chiefly, of course, because of the greater mean thickness of air space now present between the magnet poles, which reduced the mean value of $\mathfrak{I}$ in them. But what is important to our present purpose is to note that now, owing to the greater uniformity of the field, the quantity ($\mathfrak{B}$ — outside field)/4π undergoes no progressive diminution as the force rises. Table IV. gives the results. They confirm the conclusion which was provisionally stated in § 24. Here we may accept the strength of the outside field as closely approximating to the mean force within the neck, so that the first column in the table might have been styled $\mathfrak{H}$, the second last column $\mathfrak{I}$, and the last column μ.

Fig 8.

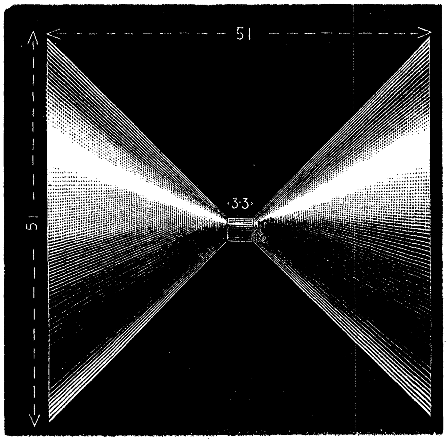


Fig. 9.

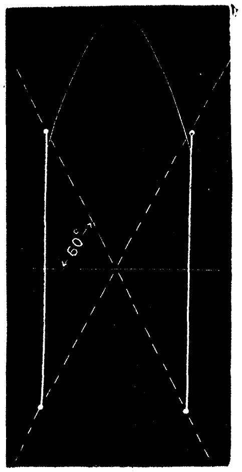


Fig. 10.

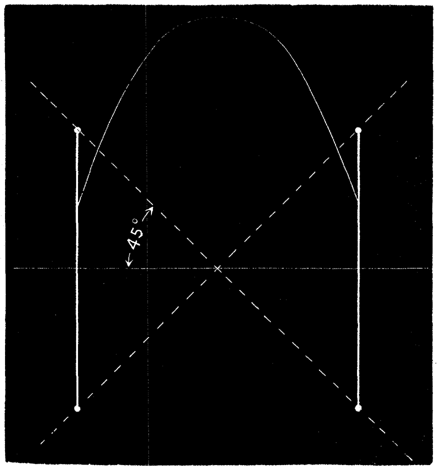


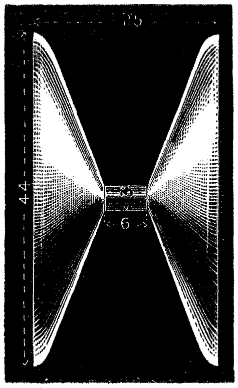
TABLE IV.—Lowmoor Wrought Iron.

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
3,080	24,130	1680	7·83
6,450	28,300	1740	4·39
10,450	32,250	1730	3·09
13,600	35,200	1720	2·59
16,390	36,810	1630	2·25
18,760	39,900	1680	2·13
18,980	40,730	1730	2·15

§ 26. It remains to describe the results of tests of other samples of wrought iron and of cast iron, of various steels, manganese steel, nickel, and cobalt. Messrs JOWITT and SONS, of Sheffield, were kind enough to supply specimens of pure Swedish iron and of steel, with which a number of experiments were made.

The following results were obtained with a bobbin of Swedish iron, described by Messrs. JOWITT as of the " L^{s}Lancash." brand, a good Swedish iron made by the Lancashire hearth process. The form and size of the bobbin are shown in fig. 11.

Fig. 11.



The cones were blunt, and their vertices were at some distance from one another, the general effect being similar to that of sharper cones with a common vertex, giving a fairly uniform, but not excessively strong, field.

TABLE V.—Swedish Iron, " L^{s}Lancash." Brand.

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1,490	22,650	1680	15·20
3,600	24,650	1680	6·85
6,070	27,130	1680	4·47
8,600	30,270	1720	3·52
18,310	38,960	1640	2·13
19,450	40,820	1700	2·10
19,880	41,140	1700	2·07

It will be noticed that the quantity in the third column, which no doubt approximates very closely to the intensity of magnetisation $\mathfrak{I}$, is practically constant, except for errors of observation. Its mean value is 1685.

§ 27. Another bobbin, described by Messrs. Jowitt as Swedish wrought iron of the celebrated Ⓛ brand, the purest and most expensive iron used in commerce, made by the Walloon process, was also turned to the shape shown in fig. 11, and tested as follows :—

Table VI.—Fine Swedish Iron, Ⓛ Brand.

Outside field.	$\mathfrak{B}$.	$\dfrac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\dfrac{\mathfrak{B}}{\text{outside field}}$.
5,310	25,670	1620	4·83
17,680	38,080	1620	2·15
19,240	39,540	1620	2·06

It would seem that the saturation value of $\mathfrak{I}$ is specifically less in this iron than in previous samples.

Cast Iron.

§ 28. Table VII., which is copied from our former paper, gives the results of tests made with a bobbin of cast iron of a form similar to that shown in fig. 1, the magnetisation being measured by reversing the bobbin in the field. The residual induction was also measured by withdrawing the bobbin, and was found to have a nearly constant value of about 400 c.g.s. units.

Table VII.—Cast Iron.

Outside field.	$\mathfrak{B}$.	$\dfrac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\dfrac{\mathfrak{B}}{\text{outside field}}$.
3,900	19,660	1250	5·04
6,400	21,930	1240	3·42
7,710	22,830	1200	2·96
8,080	23,520	1230	2·91
9,210	24,580	1220	2·67
9,700	24,900	1210	2·57
10,610	25,600	1190	2·46

§ 29. Table VIII. relates to a later test, made with Professor Tait's magnet, in which a central spindle of cast iron was used to form the neck of the bobbin, but the conical ends were of wrought iron shrunk on to the ends of the spindle. Fig. 12 shows a section of this bobbin. A similar construction has been adopted in many other cases; the use of the more permeable metal—wrought iron—for the cones has, of course, the advantage of strengthening the induction in the neck. Here the residual magnetism was measured, after the observations were otherwise complete, by slackening one of the cones, so that it might be slipped off the spindle. A suitable induction coil, wound on a ring, was then slipped on; the whole bobbin was magnetised and removed from the field, the loose end and the coil were then slipped off together, and the ballistic effect of this was observed. In these measurements the bobbin was demagnetised by the method of reversals, to get rid of the effect of previous stronger magnetisations. A similar procedure was followed in finding the residual magnetism of steel samples. The residual magnetism is allowed for in the values of 𝔅 given below.

Fig. 12.

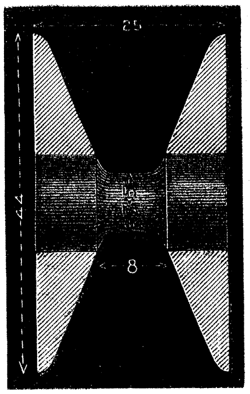


TABLE VIII.—Cast Iron.

Current in field magnets, ampères.	Outside field.	𝔅.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1·57	4,560	20,070	1230	4·40
3·62	9,120	24,630	1230	2·70
5·95	11,770	27,680	1270	2·35
8·1	13,460	28,710	1210	2·13
14·2	14,690	30,160	1230	2·05
23·0	16,200	30,920	1170	1·91
40·0	16,900	31,760	1180	1·88

These two experiments agree in assigning about 1240 as the saturation value of $\mathfrak{I}$ in this cast iron; and the apparent diminution in fields of the greatest strength is, of course, to be set down to the cause which has been fully explained in connection with wrought iron—an excess of the "outside field" over the mean force within the metal, owing to the cones being too blunt to give a very uniform field.

Steel.

§ 30. Of the following experiments, Nos. 1 to 5 were made with samples of steel supplied by Messrs. Jowitt, containing various percentages of carbon. The sample was built, in each case, into a bobbin of the form shown in fig. 13, with wrought iron cones. No. 6 was made with a specimen of Whitworth's fluid compressed steel, built with wrought iron cones into the bobbin of fig. 14. Observations were made in the strongest fields only.

Fig. 13.

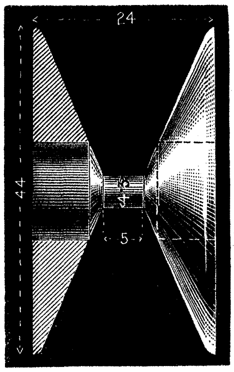


Fig. 14.

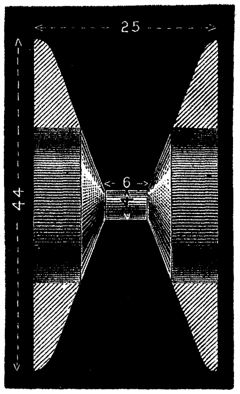


Table IX.—Steel of Various Qualities.

Description of steel.	Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1. Bessemer steel, containing about 0·4 per cent. of carbon	17,610	39,880	1770	2·27
2. Siemens-Martin steel, containing about 0·5 per cent. of carbon	18,000	38,860	1660	2·16
3. Crucible steel for making chisels, containing about 0·6 per cent. of carbon	19,470	38,010	1480	1·95
4. Finer quality of crucible steel for chisels, containing about 0·8 per cent. of carbon	18,330	38,190	1580	2·08
5. Crucible steel, containing about 1 per cent. of carbon	19,620	37,690	1440	1·92
6. Whitworth fluid compressed steel	18,700	38,710	1590	2·07

§ 31.—The following tests were made with a piece of Vickers' tool steel, built with wrought iron cones into the bobbin shown in fig. 15. In this case the magnetising field must have been sufficiently uniform to make the first column in the table represent $\mathfrak{H}$, the last μ, and the second last column $\mathfrak{I}$ very nearly. This steel had great coercive force; the residual magnetic induction (entered in the table under $\mathfrak{B}_r$) was scarcely constant, even in fields of over 10,000, and $\mathfrak{I}$ appeared to be still increasing in the strongest field to which the experiment extended.

Fig. 15.

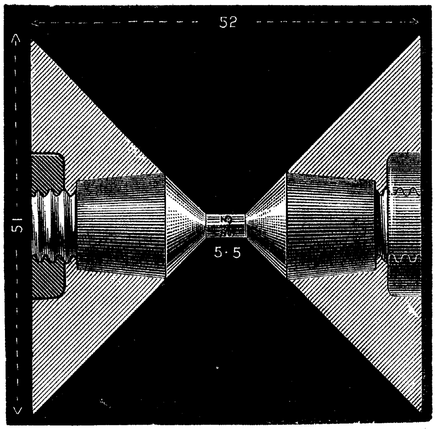


TABLE X.—VICKERS' Tool Steel.

Outside field. ($\mathfrak{H}$.)	$\mathfrak{B}_r$.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$. ($\mathfrak{I}$.)	$\frac{\mathfrak{B}}{\text{outside field}}$. ($\mu$.)
6,210	7350	25,480	1530	4·10
9,970	7670	29,650	1570	2·97
12,170	8000	31,620	1550	2·60
14,660	8030	34,550	1580	2·36
15,530	8030	35,820	1610	2·31

Taken together, the experiments on steel render it probable that there are specific differences in the saturation values of $\mathfrak{I}$ for different steels, smaller values being found in high- than in low-carbon steels. This is to be expected, in view of the decidedly low saturation value of $\mathfrak{I}$ found in cast iron.

Manganese Steel.

§ 32.—At the suggestion of Dr. J. HOPKINSON, we have examined the action of strong magnetic forces upon the remarkable alloy of iron and manganese lately introduced by Mr. R. A. HADFIELD, of Sheffield, which has many peculiar mechanical and electrical properties.* The experiments of HOPKINSON,† BOTTOMLEY,‡ and BARRETT § have shown that this steel is almost wholly destitute of magnetic suscepti-

---

* See Mr. HADFIELD's paper on Manganese Steel, 'Inst. Civ. Engin. Proc.,' February 28, 1888.

† "Magnetisation of Iron," 'Phil. Trans.,' 1885, p. 462.

‡ 'Report of the British Association for 1885,' p. 903.

§ 'Roy. Dublin Soc. Proc.,' vol. 5, 1886, p. 360.

bility. HOPKINSON found that a force $\mathfrak{H}$ of 244 produced an induction $\mathfrak{B}$ of 310, which makes the permeability only 1·27. Mr. HADFIELD was kind enough to supply us with a bar which contained about 12 per cent. of manganese and 0·8 per cent. of carbon. The metal is excessively hard, but, by raising the bar to a bright red heat and quenching it in water, it was softened sufficiently to allow pieces to be turned, with considerable difficulty, into forms suitable for these experiments.

One piece of the bar was turned into a solid bobbin, of the size and shape shown in fig. 16, and with that the following observations were made :—

Fig. 16.

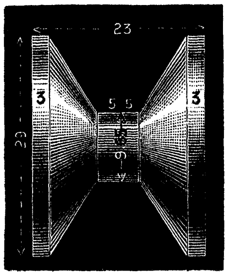
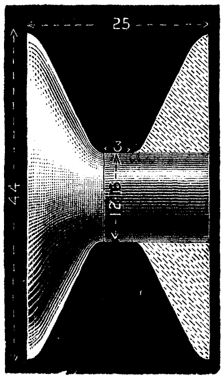


TABLE XI.—HADFIELD'S Manganese Steel.

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
2000	2770	61	1·38
3250	4560	104	1·40
3720	5090	109	1·37
4100	6010	152	1·47
5200	7320	185	1·41

Fig. 17.



§ 33. To push the induction to higher values, another bobbin was built up (fig. 17), with a central spindle cut from the bar of manganese steel, and with cones of wrought iron. The following measurements were made with it :—

Table XII.—Hadfield's Manganese Steel.

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1930	2,620	55	1·36
2380	3,430	84	1·44
3350	4,400	84	1·31
5920	7,310	111	1·24
6620	8,970	187	1·35
7890	10,290	191	1·30
8390	11,690	263	1·39
9810	14,790	396	1·51

§ 34. The figures in the last two columns of Tables XI. and XII. show as much regularity as can be expected, when it is borne in mind that they depend upon the small differences between two large quantities which had to be separately measured. The two sets of results agree well. They show that the permeability of manganese steel is, as nearly as may be judged, constant from fields of 2000 to 10,000 units, with a value approximating to 1·4 in this sample. It follows from this that, notwithstanding the excessive resistance which this material opposes to being magnetised, a respectably high intensity of magnetisation will be produced by the application of a sufficiently strong force. In the second experiment $\mathfrak{I}$ was raised to nearly 400. It is very remarkable that scarcely any of this magnetisation remains when the force is withdrawn. One might have expected that a material which resists magnetisation so strongly would possess much coercive force. In fact, however, the residual magnetism (which was determined in the second sample in the usual way, by slipping off one of the iron cones along with an induction coil) was so small that it scarcely admitted of measurement by the apparatus which served to measure the induced magnetism. After applying the strongest field the value of the residual induction $\mathfrak{B}_r$ was only about 30. It is well known that with ordinary iron and steel the magnetisation wholly, or almost wholly, disappears when the magnetising force is withdrawn, provided the force is less than a certain amount. This elastic stage in the process of magnetisation, the limits of which are exceedingly narrow in soft wrought iron, but somewhat wider in hard iron, common steels, and nickel, seems to extend, in manganese steel, up to the strongest force we have been able to apply.

Nickel.

§ 35. Two specimens of nickel, supplied by Messrs. Johnson and Matthey, have been tested. The first was cut from a bar previously used in examining the permeability of nickel when in a state of compression under the action of ordinarily weak magnetising forces.* It contained about 0·75 per cent. of iron. The bar was annealed and was built into a bobbin with wrought iron cones, and the neck was turned down to a diameter of 4 mm. (fig. 18).

Fig. 18.

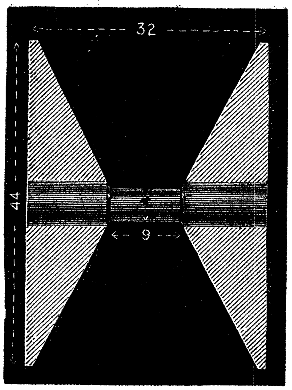


Table XIII.—Annealed Nickel (0·75 per cent. of Iron).

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
3,450	9,850	510	2·86
6,420	12,860	510	2·00
8,630	15,260	530	1·77
11,220	17,200	480	1·53
12,780	19,310	520	1·51
13,020	19,800	540	1·52

Here $\mathfrak{I}$ is sensibly constant, with a mean value of 515, of which about 160 was residual.

§ 36. The second sample was a hard-drawn wire, not annealed before testing, which contained less iron than the other (0·56 per cent.). Perhaps for this reason, the value of $\mathfrak{I}$ in it was less. The nickel formed the central spindle of a bobbin with wrought iron cones, and with a neck 5·7 mm. in diameter.

Table XIV.—Hard-drawn Nickel (0·56 per cent. of Iron)

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
2,220	7,100	390	3·20
4,440	9,210	380	2·09
7,940	12,970	400	1·63
14,660	19,640	400	1·34
16,000	21,070	400	1·32

---

* Ewing, "Magnetic Qualities of Nickel, Supplementary Paper," 'Phil. Trans.,' A, 1888.

Cobalt.

§ 37. Lastly, a piece of cobalt was tested which was cut from a cast bar supplied by Messrs. Johnson and Matthey, and turned to form the centre of a bobbin with wrought iron cones (fig. 19), and with a neck 4·48 mm. in diameter. It was found to contain 1·66 per cent. of iron.

Fig. 19.

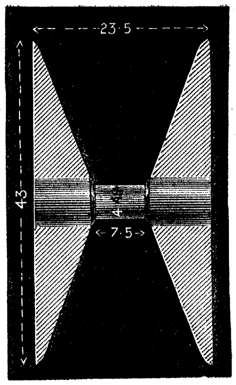


Table XV.—Cobalt.

Outside field.	$\mathfrak{B}$.	$\frac{\mathfrak{B} - \text{outside field}}{4\pi}$.	$\frac{\mathfrak{B}}{\text{outside field}}$.
1,350	16,000	1260	12·73
4,040	18,870	1280	4·98
8,930	23,890	1290	2·82
14,990	30,210	1310	2·10

It appears from this that the saturation value of $\mathfrak{I}$ for this specimen of cobalt is about 1300, or a little greater than the value we have found for cast iron. In the second, third, and fourth of these observations the residual magnetism was sensibly constant ($\mathfrak{B}_r = 1260$) ; in the first it was a little less.

---

§ 38. We may conclude that under sufficiently strong magnetising forces the intensity of magnetisation ($\mathfrak{I}$) reaches a constant, or very nearly constant, value in wrought iron, cast iron, most steels, nickel, and cobalt. The magnetic force at which $\mathfrak{I}$ may be said to become practically constant is less than 2000 c.g.s. units for wrought iron and nickel, and less than 4000 for cast iron and cobalt. In stronger fields, the relation of magnetic induction to magnetic force may be expressed by the formula

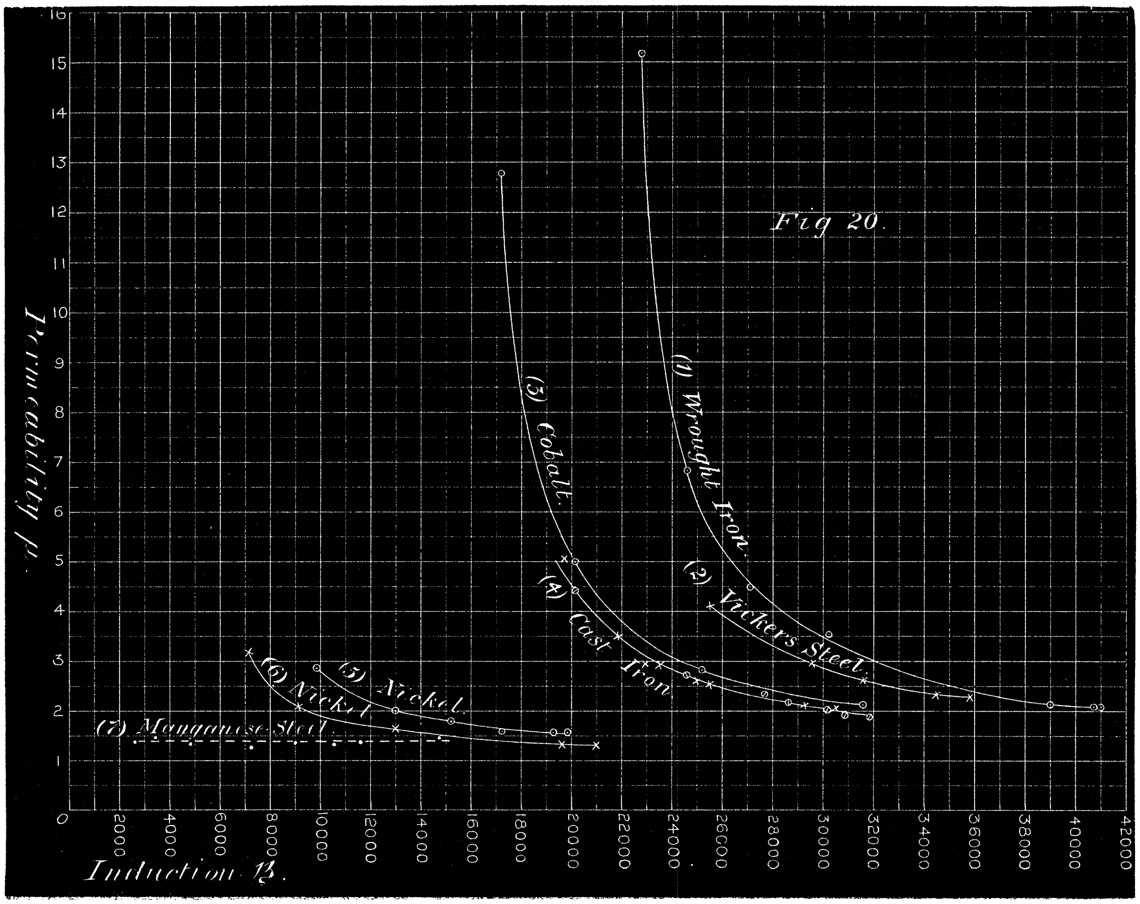
$$\mathfrak{B} = \mathfrak{H} + \text{constant}.$$

For the particular specimens we have tested, the value of this constant ($4\pi\mathfrak{I}$) is about

21,360 in wrought iron, 15,580 in cast iron, 5030 and 6470 in nickel, and 16,300 in cobalt.

The experiments give a definite meaning to the term "saturation," as applied to magnetic state. When magnetisation is measured by the induction 𝔅, the term saturation is inapplicable; there is apparently no limit to the value to which the induction may be raised. But, when we measure magnetisation by the intensity of magnetism ℑ, we are confronted with a definite limit—a true saturation value, which is reached or closely approached by the application of a comparatively moderate magnetic force. There is nothing to show that the approach to this limit is not asymptotic; but in wrought iron it is practically reached before the magnetic force rises to 2000 c.g.s., and after that a ten-fold increase in the force produces no material change in the intensity of magnetism.

Fig. 20.



§ 39.—The results are further summarised in fig. 20, in which Rowland's curve, showing the relation of the permeability μ to the induction 𝔅, is drawn from the data supplied by the experiments on—

(1) Swedish wrought iron (Table V.).
(2) VICKERS' tool steel (Table X.).
(3) Cobalt (Table XV.).
(4) Cast iron (Tables VII. and VIII. The points taken from Table VII. are marked thus, ×, and those from Table VIII. thus, ⦶.).
(5) Annealed nickel with 0·75 per cent. of iron (Table XIII.).
(6) Hard-drawn nickel with 0·56 per cent. of iron (Table XIV.).
(7) HADFIELD'S manganese steel (Table XII.).

If the magnetic force $\mathfrak{H}$, instead of the induction $\mathfrak{B}$, had been taken as abscissa, the curves (with the exception of those relating to VICKERS' steel and manganese steel) would have sensibly lain upon rectangular hyperbolas with $\mathfrak{H} = 0$ and $\mu = 1$ for asymptotes.